Is Regulation Biasing Risk Management?

Financial Regulation: More Accurate Measurements for Control Enhancements and the Capture of the Intrinsic Uncertainty of the VaR

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Preliminary Statements

1. <u>Risk management moto: *Si Vis Pacem Para Belum*</u>
    1. Awareness
    2. Prevention
    3. Control
    4. Mitigation

2. <u>A risk is not a loss.</u>
    1. It may never crystallise
    2. The caracteristics are assumptions

3. <u>Modelling is not the truth</u>
    1. An exact replication of the Univers with mathematics is Utopia
    2. A model defines itself by its limitations.

4. <u>The couple Risk-Return is fundamental</u>

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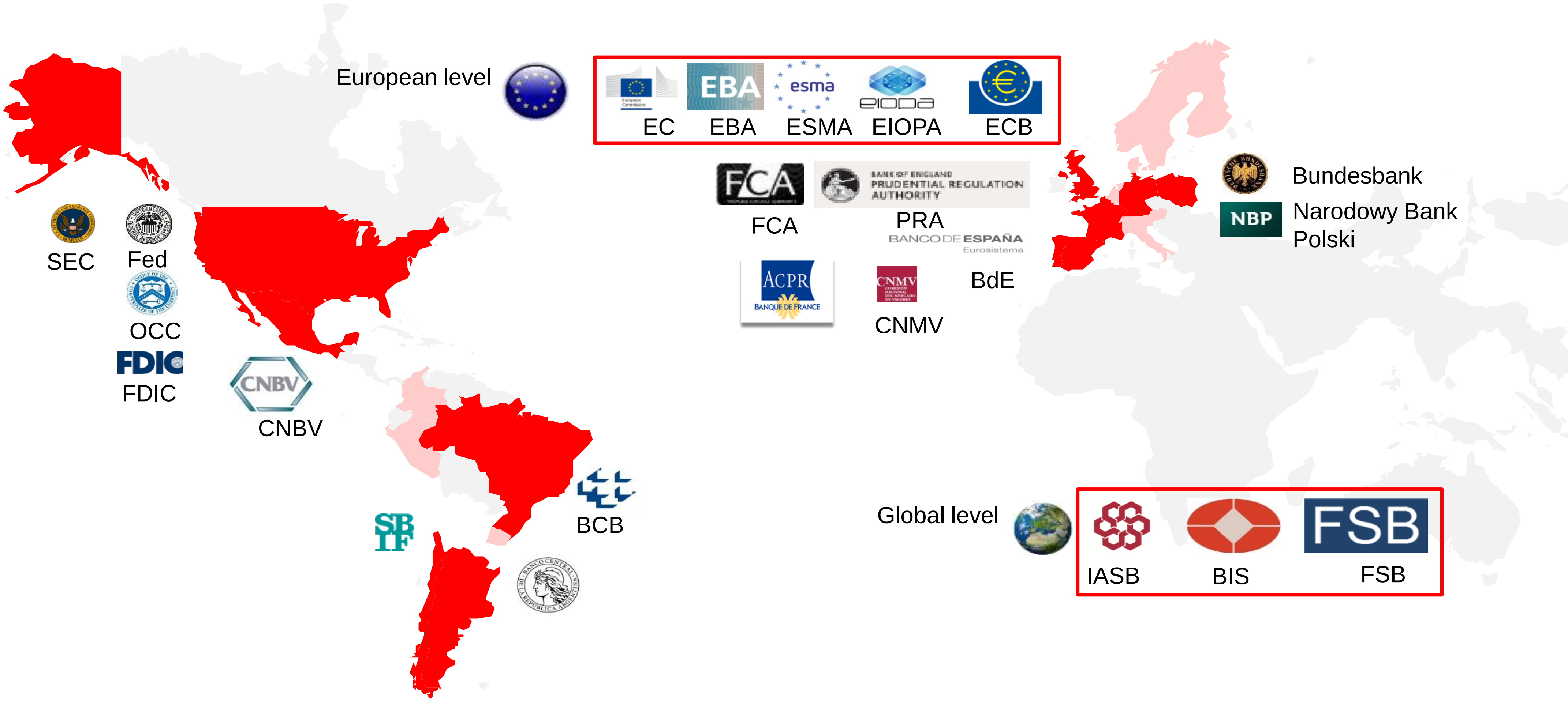
Regulatory environment considered

KEY REGULATORS AND SUPERVISORS



Regulatory Statements

- **Risk Measures in the Regulation:**
  - Market Risk: VaR 95% historical or Guassian (traditionally). Moving toward Expected Shortfall
  - Credit Risk: Percentile at 99% used. LGD is an expected shortfall (spectrum). Usually a logistic regression for the Probability of Default.
  - Counterparty: Expected Positive Exposure (EPE). Gaussian assumption.
  - Operational Risk: VaR at 99.9% for the regulatory capital and 99.95-8 for the economic capital. Any distribution could be used.
  - Stress Testing: Stress VaR, etc. A lot more latitude though?

- **False Statements?**
  - Stability of the risk measures when data are not stationary?
  - Data sets selected? 5y, 10y?
  - VaR non sub-additive – ES sub-additive?
  - VaR not capturing tail information?
  - Empirical distribution not conservative enough?

Problematic

1. For a single kind of risk (univariate): the choice of the level of confidence is not determining, while the distribution is….?

2. For multiple kind of risks (multivariate): for which combination of distributions is the sub-additivity property fulfilled ?

    - Are the model reliable to evaluate these risk measures?

3. Given that each risk may be modelled considering different distributions and using different confidence level for the risk measure, what is the impact of the non sub-additivity?

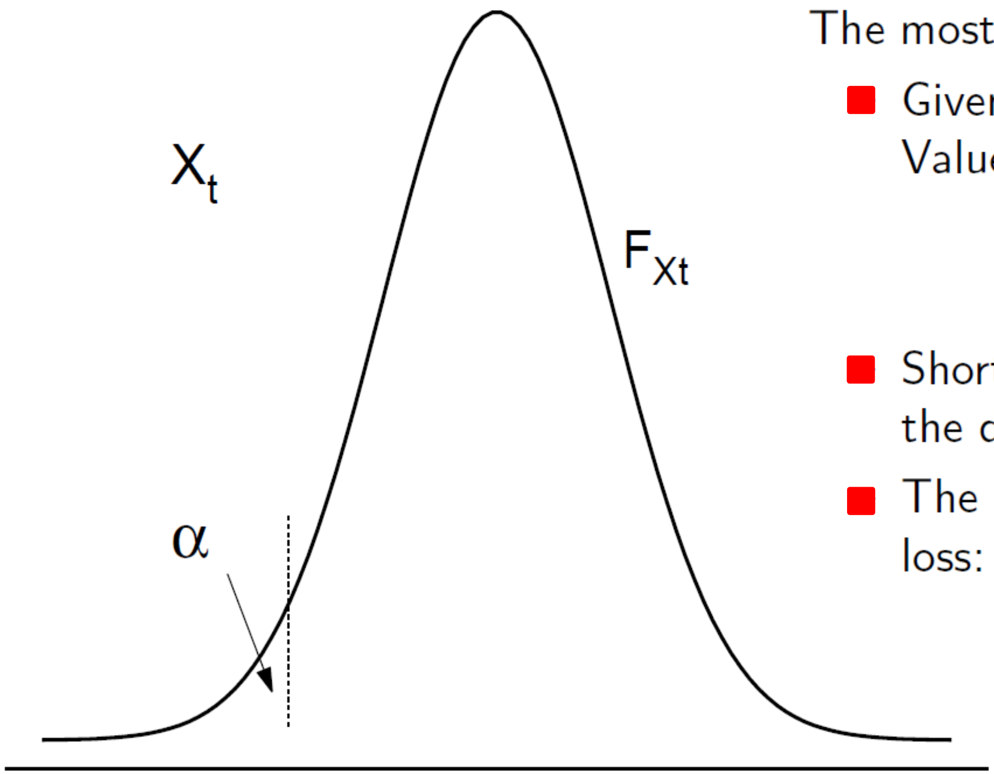
4. Is that more efficient in terms of risk management to measure the risk and then build a capital buffer or to adjust the risk taken considering the capital we have? (Inverse problem)

5. The previous points are all based on uni-modal parametric distributions, what is the impact of using multimodal distributions in terms of risk measurement and management?

    - How can we combine the various risks to obtain a holistic metric?
    - Can we combine various risk measures evaluated at different confidence level?

Once applied, is the concept of risk measure still meaningful?

Risk Measurement in a nutshell



The most well-known risk measure is the Value at Risk (VaR).

- Given a position X and a number $\alpha \in [0, 1]$, we define the Value at Risk (VaR) at confidence level α, by

$$Pr[X \geq VaR_{\alpha}] = 1 - \alpha$$

- Shortly speaking we compute A QUANTILE: it depends on the distribution and the choice of the level.
- The Expected Shortfall (ES) is the expected worst possible loss:

$$ES_{\alpha}(X) = E[X|X \geq VaR_{\alpha}]$$

Though one may wonder……

1. What is the role of the distributions fitted to each factor?

2. What is the impact of the level?

3. What is the interest to use the ES if VaR is subadditive?

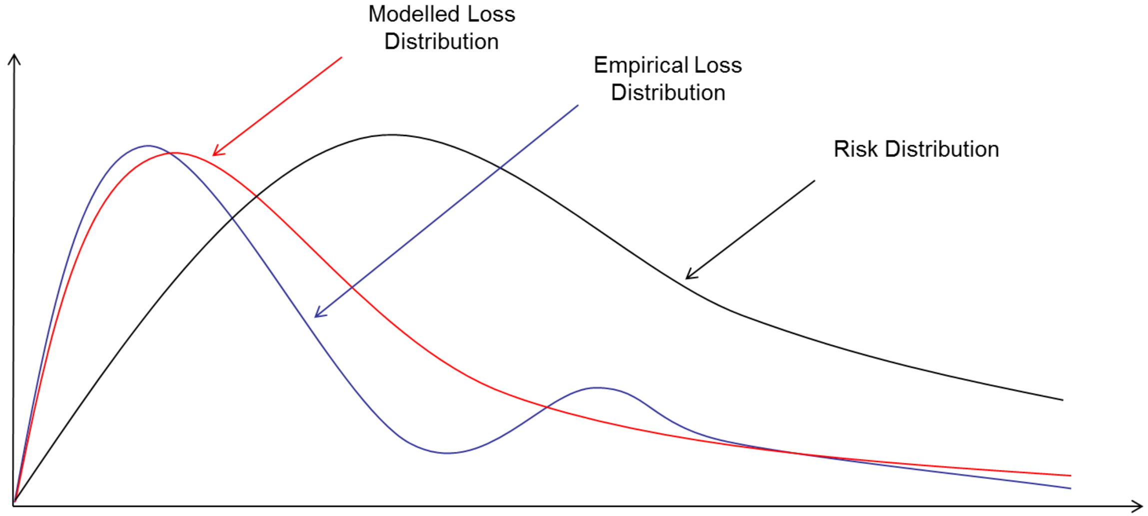
4. What is the sense to aggregate risks which are not computed at the same level

5. What are the objective behind the demands of regulators?

6. Sub additivity? - Conservatism? - Capital?

Experimentation

- **Distributions**
  - **Empirical, lognormal, Weibull, GPD, GH, Alpha-stable, GEV**
  - **Parameterization: MLE, Hill, Block Maxima**
  - **Goodness-of-fit: KS, AD**
- **Risk Measures**
  - **ES**
  - **VaR**
  - **Spectral**
  - **Distortion**
- **Data**
  - **Market data (Dow Jones)**
  - **Operational Risk data (EDPM)**



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VaR vs ES?

Table 1
09-14

Distribution / %tile	Empirical VaR	Empirical ES	LogNormal VaR	LogNormal ES	Weibull VaR	Weibull ES	GPD VaR	GPD ES	GH VaR	GH ES	Alpha-Stable VaR	Alpha-Stable ES	GEV VaR	GEV ES	GEVbm VaR	GEVbm ES
90%	575	2 920	686	2 090	753	1 455	10 745	146 808	627	2 950	582	29 847 $+\infty$	2.097291e+06	$+\infty$	626	12 755
95%	1 068	5 051	1 251	3 264	1 176	1 979	19 906	224 872	1 317	5 006	1 209	58 872 $+\infty$	2.869971e+07	$+\infty$	1 328	24 616
97.5%	1 817	8 775	2 106	4 925	1 674	2 572	37 048	368 703	2 608	8 187	2 563	116 016 $+\infty$	3.735524e+08	$+\infty$	2 762	47 356
99%	3 662	18 250	3 860	8 148	2 439	3 468	84 522	758 667	5 917	14 721	7 105	283 855 $+\infty$	1.073230e+10	$+\infty$	7 177	111 937
99.9%	31 560	104 423	13 646	24 784	4 852	6 191	675 923	5 328 594	28 064	46 342	98 341	2 649 344 $+\infty$	4.702884e+13	$+\infty$	77 463	945 720

Table 2
09-11

Distribution / %tile	Empirical VaR	Empirical ES	LogNormal VaR	LogNormal ES	Weibull VaR	Weibull ES	GPD VaR	GPD ES	GH VaR	GH ES	Alpha-Stable VaR	Alpha-Stable ES	GEV VaR	GEV ES	GEVbm VaR	GEVbm ES
90%	640	2 884	738	2 238	803	1 542	11 378	137 154	686	2 925	625	15 237 $+\infty$	676	10 130 $+\infty$	2 557	74 000 989 $+\infty$
95%	1 086	4 965	1 344	3 492	1 249	2 093	20 994	204 873	1 432	4 874	1 283	29 652 $+\infty$	1 438	19 292 $+\infty$	7 414	147 997 711 $+\infty$
97.5%	1 953	8 387	2 259	5 265	1 771	2 714	38 938	327 418	2 788	7 784	2 684	57 580 $+\infty$	2 999	36 538 $+\infty$	21 053	295 983 188 $+\infty$
99%	4 006	16 993	4 133	8 693	2 570	3 639	88 471	652 229	6 014	13 442	7 311	137 784 $+\infty$	7 821	84 287 $+\infty$	82 476	739 897 783 $+\infty$
99.9%	30 736	86 334	14 561	26 378	5 078	6 384	700 863	4 182 440	24 409	37 774	96 910	1 184 005 $+\infty$	85 225	658 076 $+\infty$	2 494 850	7 395 636 153 $+\infty$

Table 3
12-14

Distribution / %tile	Empirical VaR	Empirical ES	LogNormal VaR	LogNormal ES	Weibull VaR	Weibull ES	GPD VaR	GPD ES	GH VaR	GH ES	Alpha-Stable VaR	Alpha-Stable ES	GEV VaR	GEV ES	GEVbm VaR	GEVbm ES
90%	501	3 002	568	1 722	637	1 246	9 269	121 928	494	3 006	504	336 621 $+\infty$	504	6 578	2 063	3 009 981 $+\infty$
95%	906	5 344	1 034	2 688	1 002	1 702	17 197	176 742	1 035	5 308	1 060	672 418 $+\infty$	1 044	12 442	6 021	6 016 460 $+\infty$
97.5%	1 397	9 625	1 739	4 057	1 435	2 222	31 868	275 836	2 129	9 162	2 272	1 343 100 $+\infty$	2 122	23 416	17 206	12 022 771 $+\infty$
99%	2 967	21 048	3 182	6 702	2 103	3 003	71 958	537 693	5 186	18 043	6 380	3 351 534 $+\infty$	5 345	53 611	67 972	30 005 581 $+\infty$
99.9%	32 977	145 966	11 211	19 917	4 232	5 387	556 192	3 405 540	36 582	74 465	90 939	33 319 684 $+\infty$	53 329	411 769	2 099 862	297 164 627 $+\infty$

Univariate Risk Measures - This table exhibits the VaRs and ESs for the height types of distributions considered - empirical, lognormal, Weibull, GPD, GH, -stable, GEV and GEV fitted on a series of maxima - for five confidence level (90%, 95%, 97.5%, 99% and 99.9%) evaluated on the period 2009-2014.

Relationship between VaR and ES

- Gaussian distribution:

$$ES_\alpha(X) = c_1 + c_2 e^{c_3(VaR_\alpha - c_1)^2},$$

$$c_1 = \mu,\ c_3 = -\frac{1}{2\sigma^2},\ c_2 = \frac{\sigma}{(1-\alpha)\sqrt{2\pi}}.$$

- Generalized Hyperbolic distribution:

$$ES_\alpha(X) = c_1 + c_2 + c_3 e^{c_4(VaR_\alpha - c_1)^2},$$

$c_1 = \mu,\ c_4 = -\frac{1}{2\sigma^2},\ c_3 = \frac{\sigma}{(1-\alpha)\sqrt{2\pi}} E[\sqrt{W}],\ c_2 \gamma E[W]$ where W is a Generalized Inverse Gaussian distribution.

- Generalized Pareto distribution:

$$ES_\alpha(X) = c_1 + c_2 VaR_\alpha(X),$$

$$c_1 = \frac{\sigma_\xi u}{1-\xi},\ c_2 = \frac{1}{1-\xi}.$$

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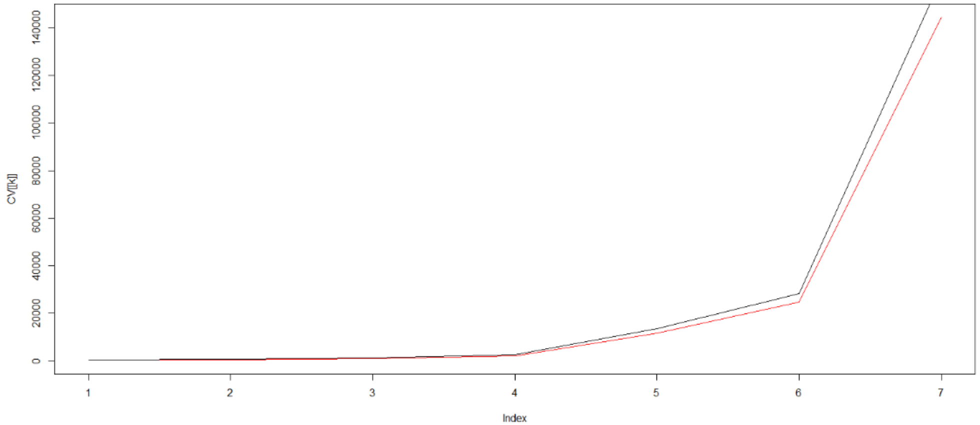
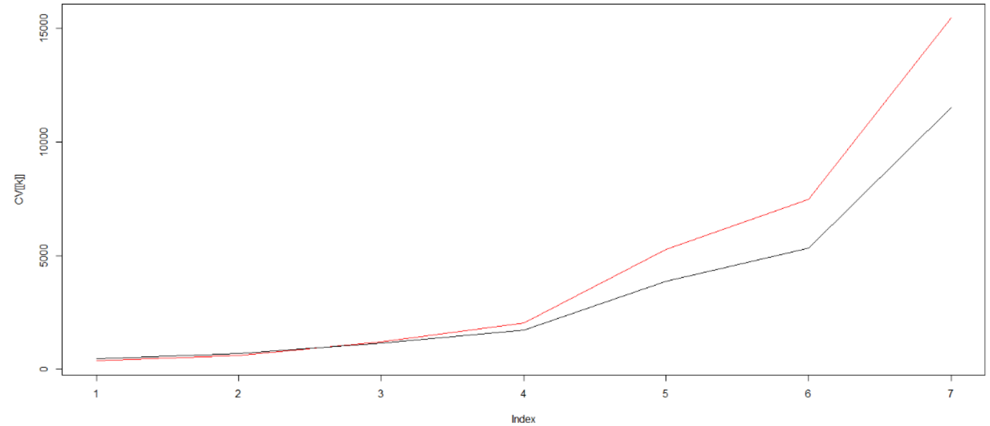
Spectrum Conundrum……… VaR(X + Y) vs VaR(X) + VaR(Y)

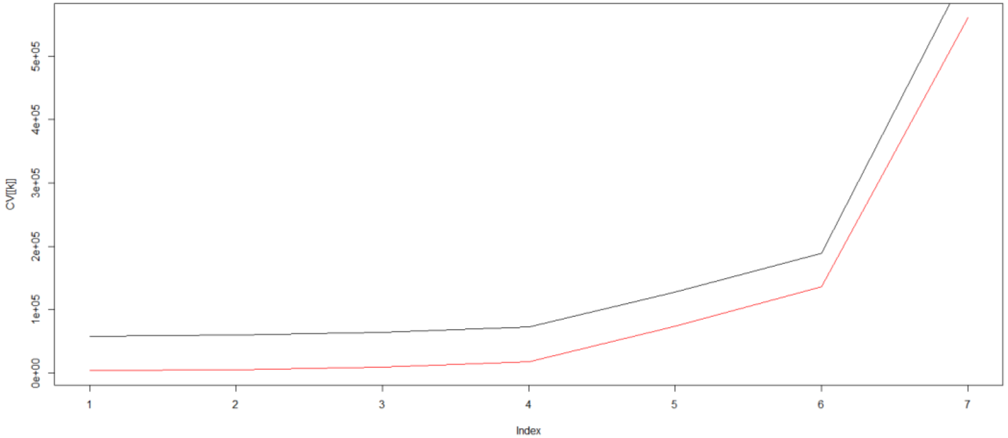
Alpha-Stable GEV



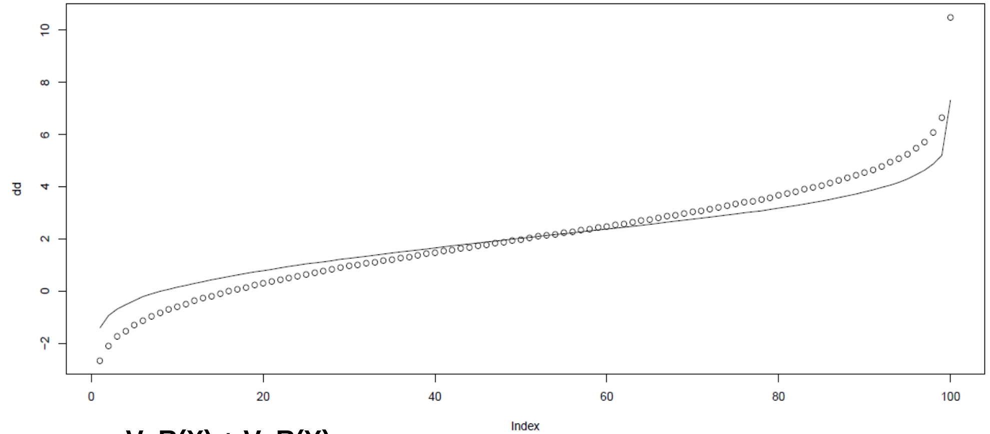
lognormal Weibull



GPD Weibull



Gaussian Benchmark



VaR(X) + VaR(Y)

VaR(X+Y)

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With value…..

LN-LN 1	393	663	1, 373	2, 503	7, 721	11, 661	27, 292
LN-LN 2	395	667	1, 376	2, 503	7, 721	11, 677	27, 517
LN-WE 1	447	742	1, 439	2, 427	6, 299	8, 924	18, 498
LN-WE 2	564	826	1, 374	2, 068	4, 654	6, 406	14, 066
LN-GPD 1	4, 321	6, 181	11, 432	21, 158	88, 382	163, 788	689, 569
LN-GPD 2	58, 968	60, 766	65, 759	74, 945	138, 510	209, 859	726, 643
LN-GH 1	364	611	1, 313	2, 569	9, 882	16, 037	41, 329
LN-GH 2	480	742	1, 418	2, 528	8, 205	12, 765	30, 592
LN-AS 1	377	614	1, 269	2, 461	10, 965	21, 402	111, 987
LN-AS 2	476	725	1, 374	2, 472	9, 657	18, 319	101, 929
LN-GV 1	25, 132	137, 464	2, 097, 977	28, 700, 959	10.73e9	134.51e9	47, 029e9
LN-GV 2	25, 313	138, 221	2, 095, 098	29, 156, 891	10.47e9	135.38e9	45, 501e9
LN-GVb 1	366	614	1, 312	2, 579	11, 037	20, 542	91, 109
LN-GVb 2	481	742	1, 423	2, 571	9, 670	17, 603	80, 694

Table 5: The sum of VaR(X) and VaR(Y) (line 1) versus VaR(X + Y) (line 2) for couple of distributions: LN = lognormal, WE = Weibull, GPD = Generalised Pareto, GH = Generalised Hyperbolic, AS = Alpha-Stable, GV = Generalised Extreme Value, GVB = Generalised Extreme Value calibrated on maxima. The percentiles represented are the 70th, 80th, 90th, 95th, 99th, 99.5th and 99.9th.

Sub-additive or not?

1. VaR is known to be sub-additive (Degen and Embretchtz, 2007: A risk measure $\rho(.)$ is sub-additive if $\rho(X + Y) \leq \rho(X) + \rho(Y)$:

    1. for stable distribution,
    2. for all log-concave distribution,
    3. for the infinite variance stable distributions with finite mean
    4. for distribution with Pareto type tails when the variance is finite.

2. The non-sub-additivity of VaR can occur

    1. when assets in portfolios have very skewed loss distributions;
    2. when the loss distributions of assets are smooth and symmetric,
    3. when the dependency between assets is highly asymmetric, and
    4. when underlying risk factors are independent but very heavy-tailed.

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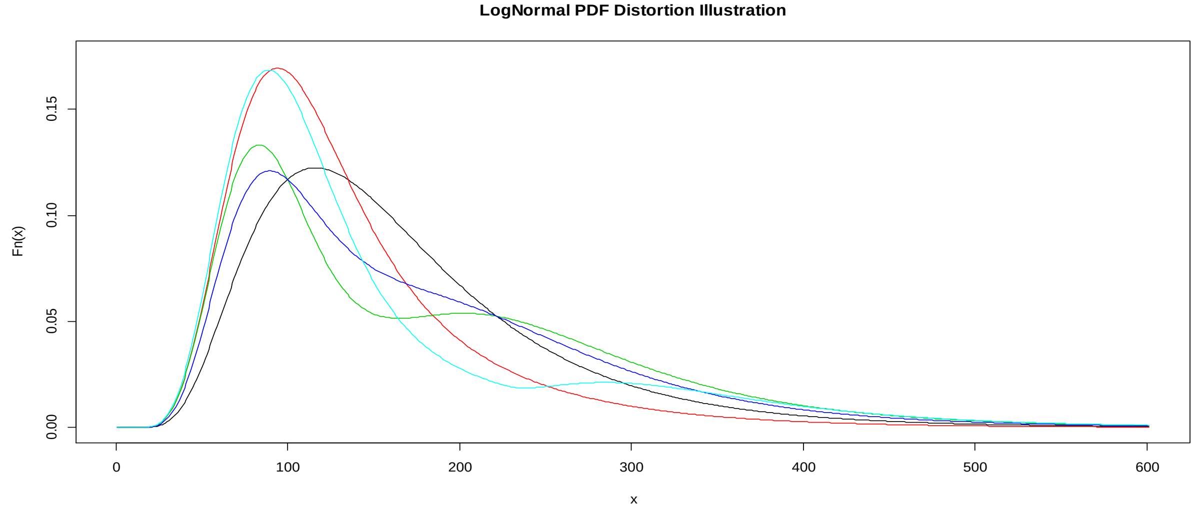
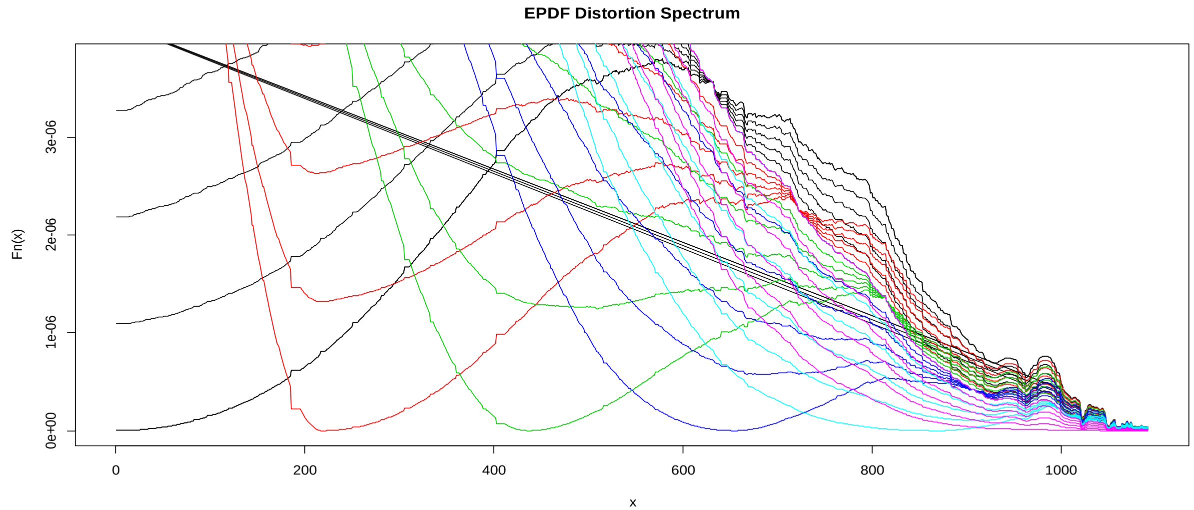
Distortion Risk Measures (1/1)

Impact on a parametric distribution



Impact on a non-parametric distribution



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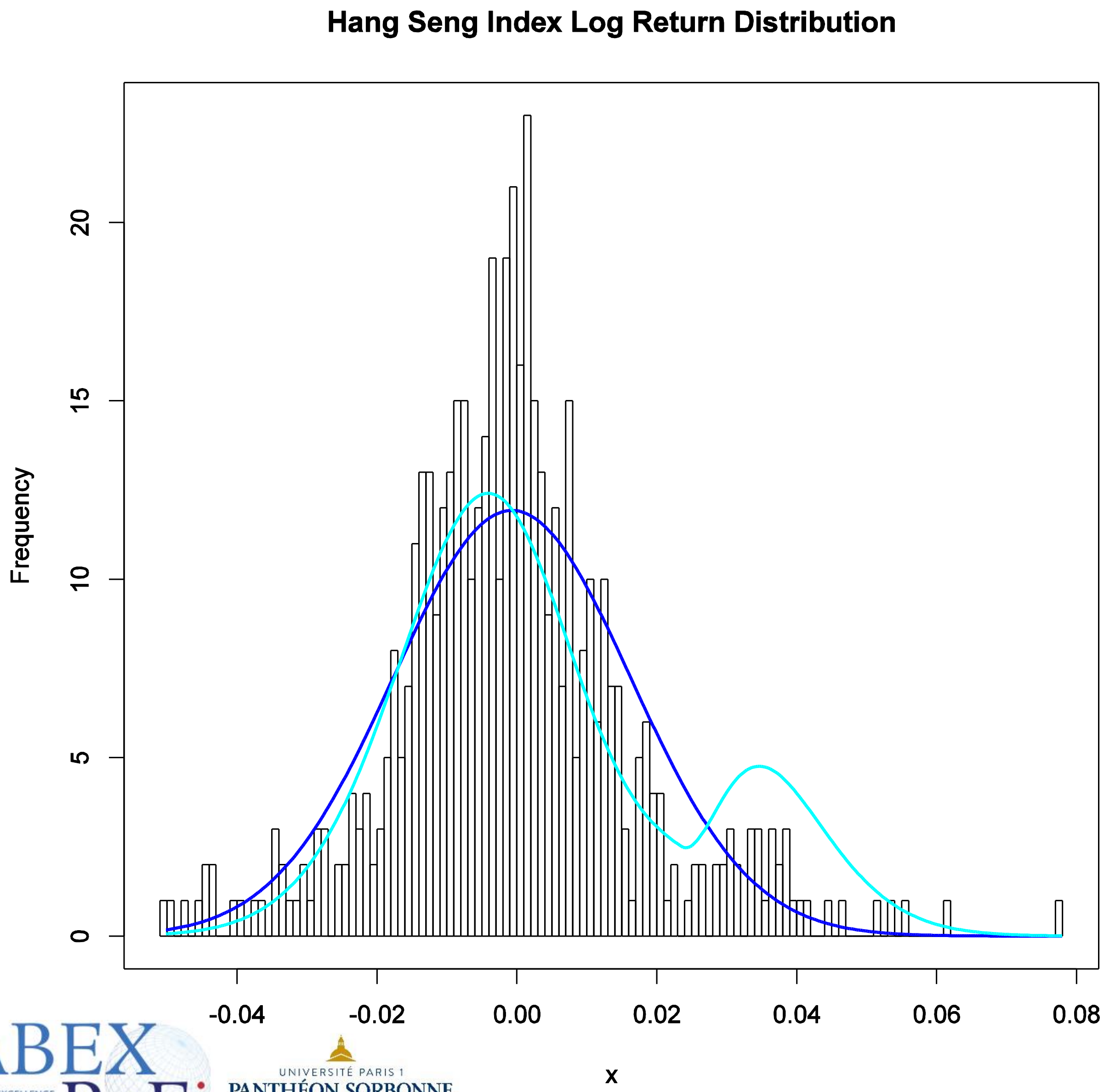
Distortion Risk Measures (2/2)



HangSend Application:

- **The risk is much lower than the one captured with a Gaussian distribution**
- **The potential regulatory capital might be lower**
- **The mitigants/ hedging strategies can be biased if relying on inapropriate measure**

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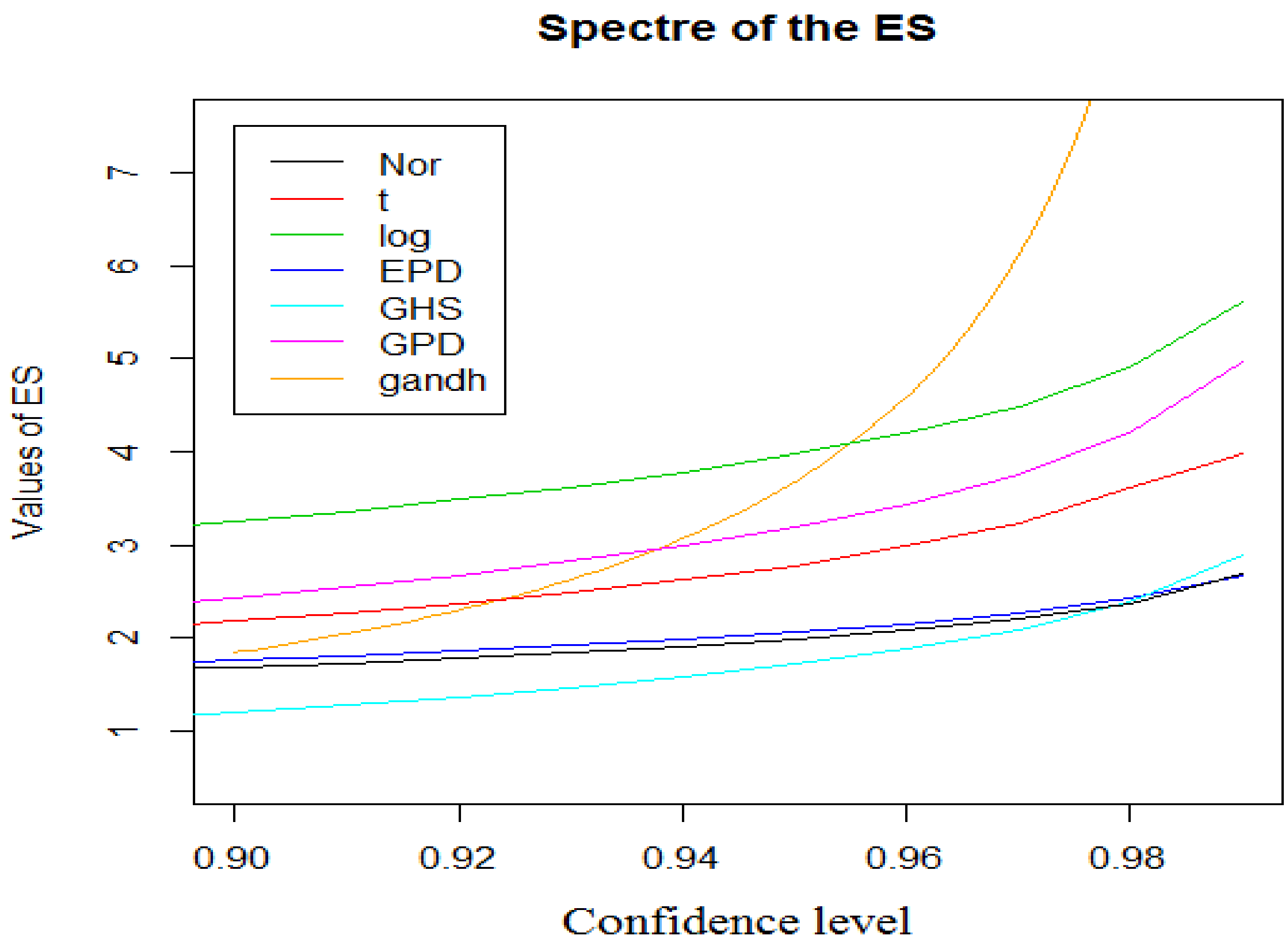
Spectral measure versus spectrum

- Spectral measure is "a kind " of aggregation (EX : ES) . It provides a value. The aggregation can have no sense (role of the confidence level p.

- Thus the use of several confidence levels p_i ; $i = 1;\ldots;k$ allowing to have a spectrum representation of the risk measure (VaR or ES) could be interesting.

- The limited approach proposed by the regulator which mixes distribution and confidence level is questionable: The spectrum of a risk Measure permits to appreciate the real influence of the levels for a given distribution, to analyse theabrupt changes in the risks and to have a clear idea of the changes of the subadditivity property for the VaR.

Spectral measure versus spectrum: ES illustration



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Interesting Behaviour

-0.012	0.022	-0.013	-0.018	-0.015	-0.031	-0.020	-0.026
-0.038	**0.011**	0.028	0.023	0.022	0.024	0.044	0.073
0.074	0.080	0.139	0.144	0.194	0.171	0.167	0.163
0.142	0.141	0.134	0.150	0.179	0.175	0.105	0.107
0.016	**-0.001**	-0.002	-0.003	0.013	-0.021	-0.048	-0.011
-0.016	**0.045**	0.074	0.032	0.074	0.166	0.124	0.104
0.098	0.019	**-0.037**	-0.079	-0.100	-0.120	-0.144	-0.047
-0.070	-0.086	-0.136	-0.234	-0.291	-0.352	-0.272	-0.197
-0.098	0.038	0.121	-0.313	-0.299	-0.483	-0.621	-0.422
-0.457	**0.099**	0.272	0.381	0.430	0.656	0.754	0.533
0.693	1.035	0.715	1.087	0.778	**−0.167**	-0.479	-0.522
-0.759	-3.391	-2.265	-4.190	-3.137	-6.484	-1.975	**9.502**
6.873	16.636	69.495	50.091	7,118.689	8,798.144	**−148,979.500**	NA

Table 15: This table shows the differences between the sum VaR(X) and the VaR(Y) and the VaR(X + Y). The random variable X and Y have been obtained on 2 identical GEV distribution. When the values are positive, the VaR is sub-additive, when the values are negative the VaR is not. The turning points are highlighted in bold. The percentiles represented are sequentially going form 1% to 99% by 1%, and to capture the tail, the 99.95th, 99.9th, 99.95th and 99.99th percentiles are added.(49)

An area as a risk measure and an alert indicator

- In all previous approaches, we always work with a point estimation of the VaR. We know that mainly all point estimation can be biased.

- A natural way would be to use a confidence interval around this estimate and to derive another way to compute the capital charge. We would obtain an upper bound and a lower bound that could be discussed with the regulators.

Estimation of the VaR

- Given a sequence of n r.v. $X_1, ..., X_n$, We rank them $X_{(1)} \leq ... \leq X_{(n)}$ and an estimate of VaR is:

$$\widehat{VaR_p} = X_{(m)} \qquad (1)$$

where $0 < p < 1$, $m = np$ if np is an integer; $m = [np] + 1$ otherwise

- The computation is obtained using the unknown c.d.f F_θ of the set of r.v. In the following, we denote its estimate $F_{\hat{\theta}}$.
- The point estimate of $\widehat{VaR_p}$ from $X_1, ..., X_n$ can be far or close to the true VaR.

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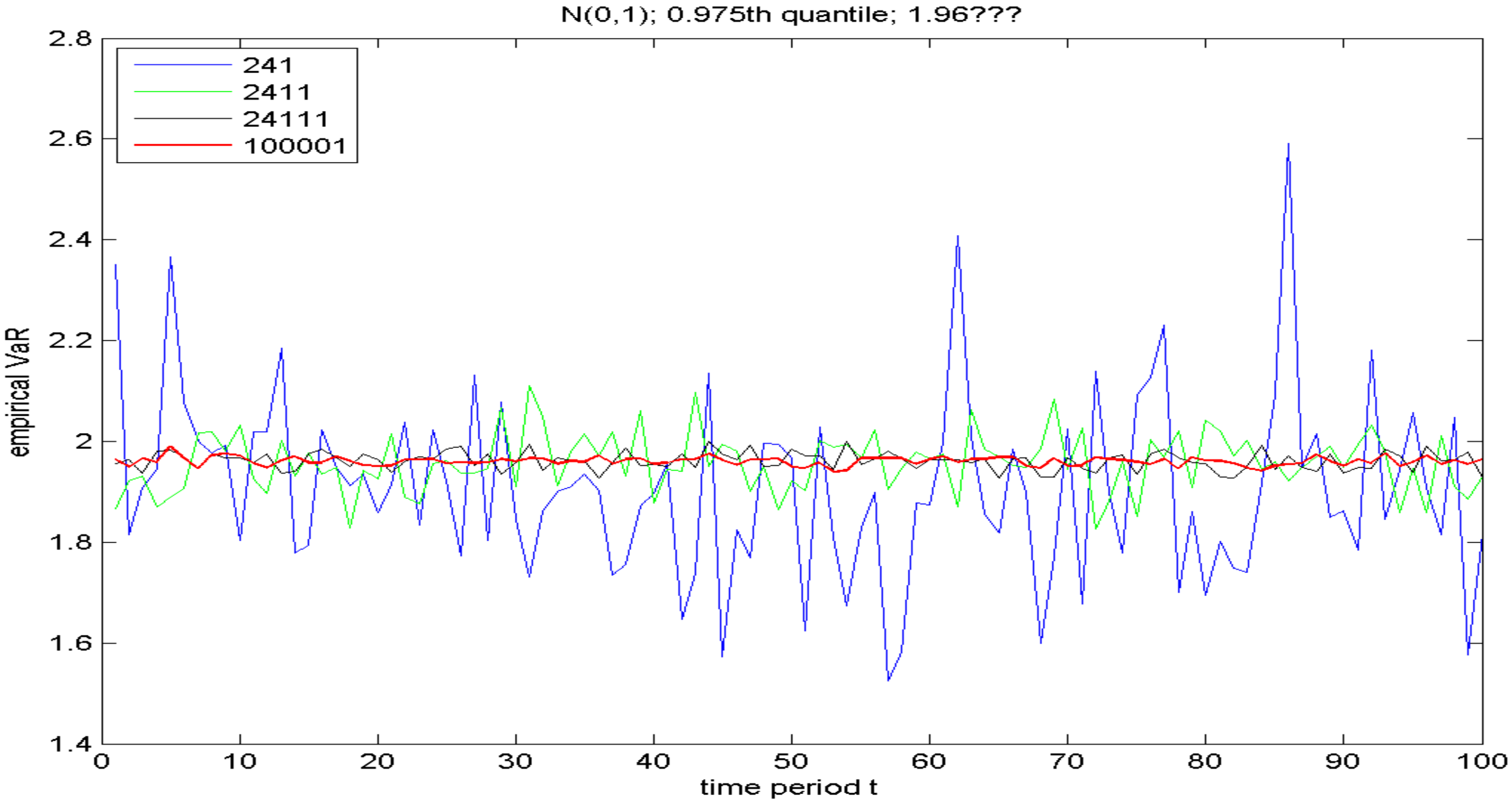
Example 1: Fθ is Gaussian



An unique realization of $X_{(m)}$ is not sufficient to have a robust risk measure.

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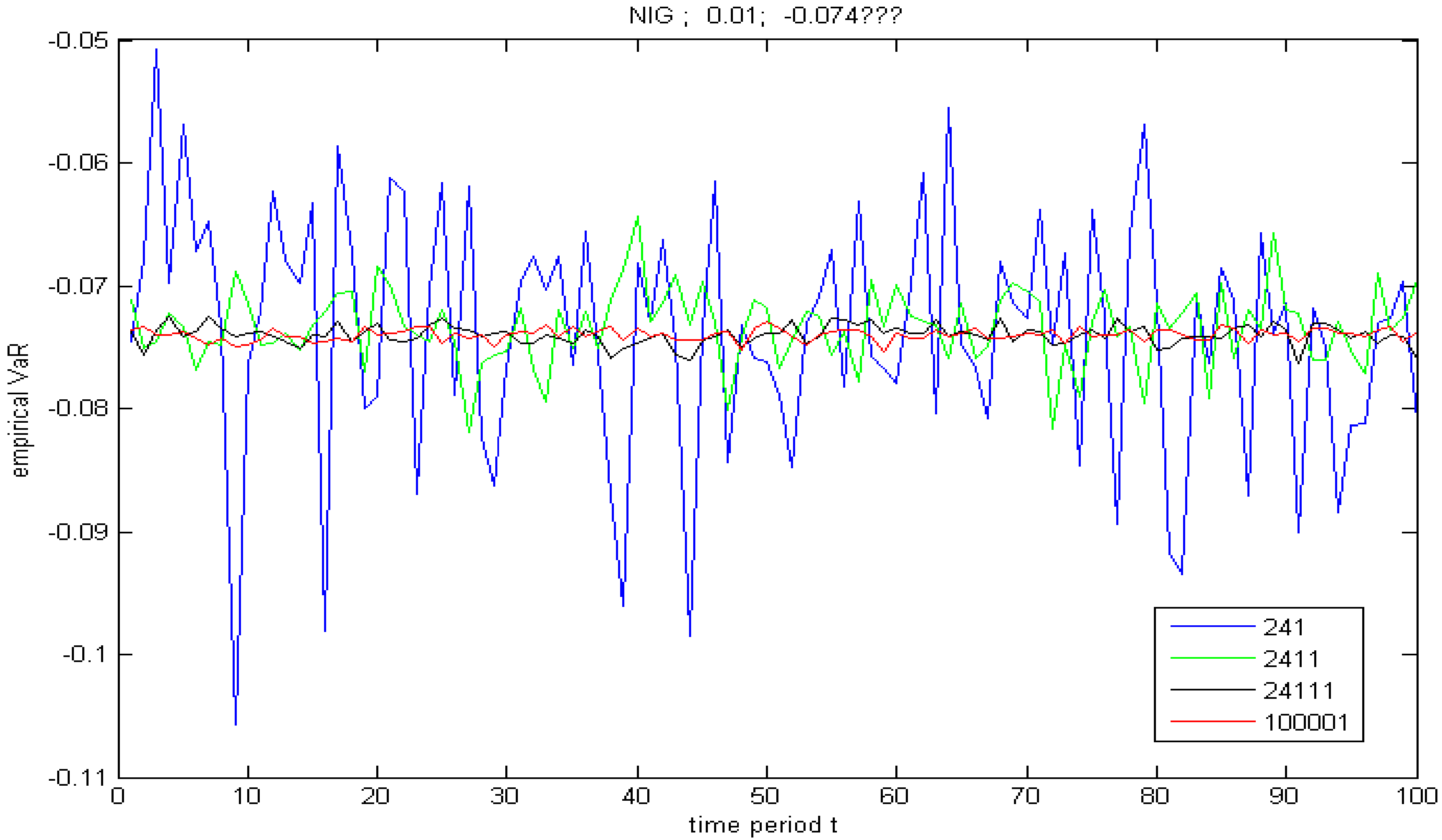
Example 2: Fθ is a NIG



Properties of $X_{(m)}$

- $X_{(m)}$ is known to be a consistent estimator of VaR_p (Serfling, 2001).
- Smirnov (1949) proves that the asymptotic distribution of $X_{(m)}$ is Gaussian under smooth conditions on F_θ (intuition CLT).
- Zhu and Zhou (2009) provides another asymptotic distribution of $X_{(m)}$ (intuition Saddlepoint approach).

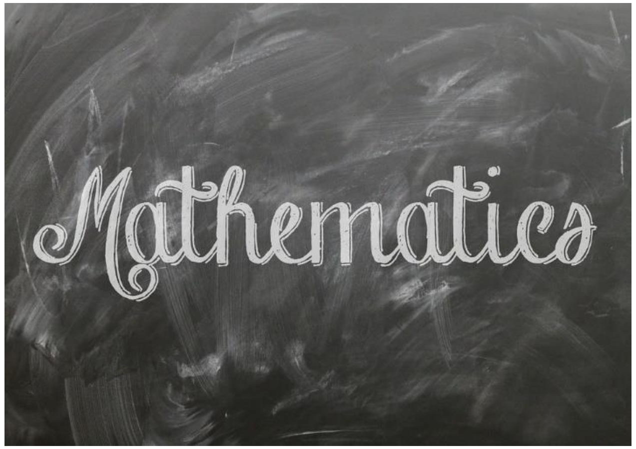
We can use these two results to build a CI around VaR_p:

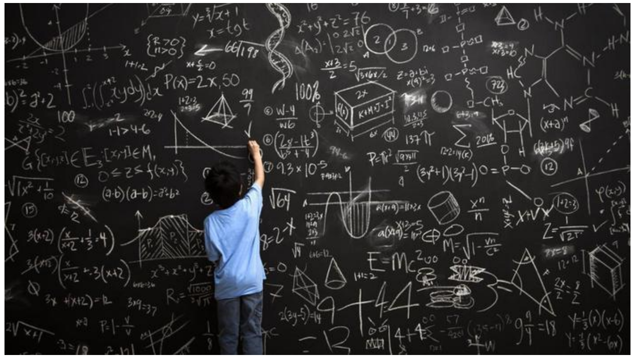
$$\left[X_{(m)} - z_{1-\frac{q}{2}}\sqrt{\hat{V}},\quad X_{(m)} - z_{\frac{q}{2}}\sqrt{\hat{V}}\right] \tag{2}$$

- This interval is symmetric.

$$\left[X_{(m)} - Z_{1-\frac{q}{2}},\quad X_{(m)} - Z_{\frac{q}{2}}\right] \tag{3}$$

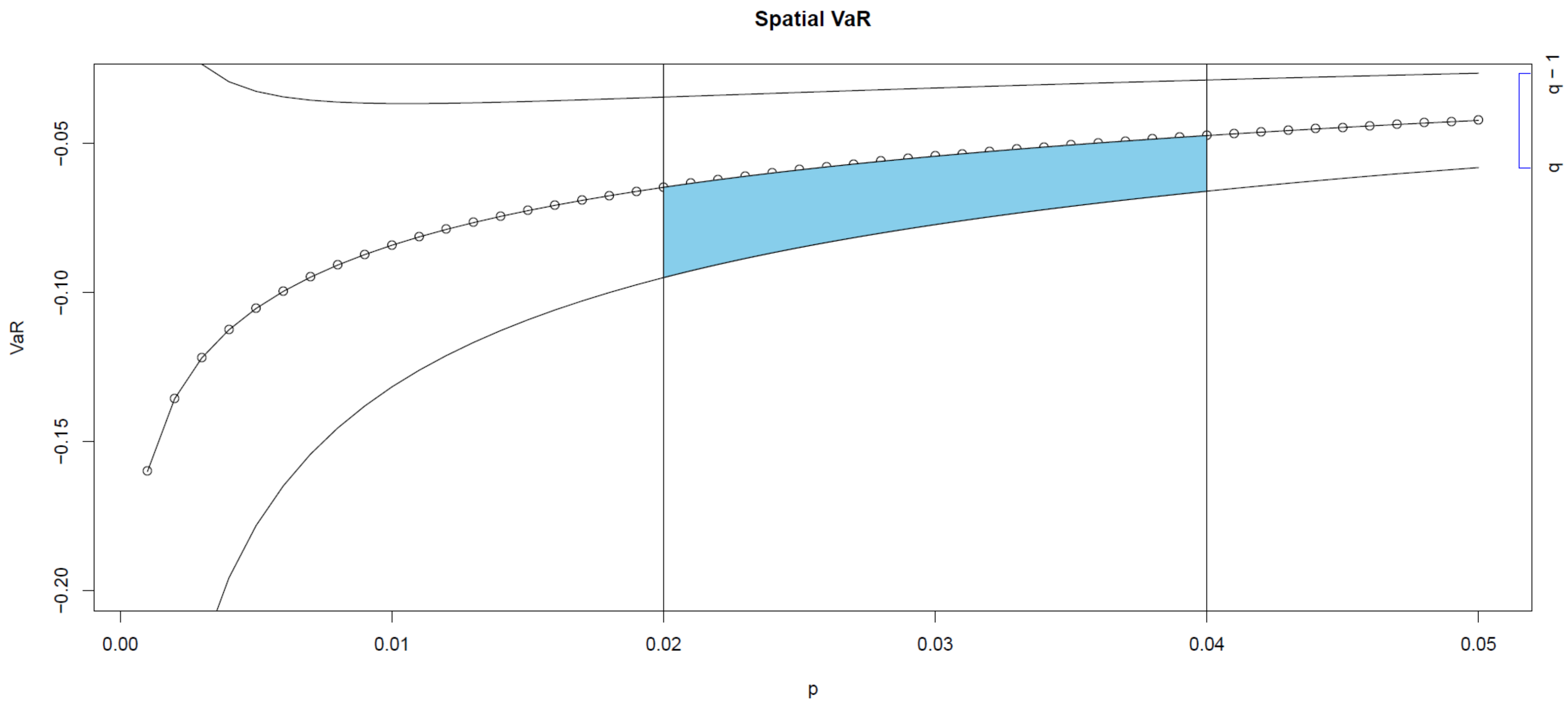
$Z_q = \Psi_q^{-1}(\sqrt{n}\widehat{\omega}^\sharp)$. Ψ and $\widehat{\omega}^\sharp$ are provided in expressions (6) and (7). This interval can be symmetric or asymmetric





Spatial VaR (Spectrum Stress VaR)

The figure exhibits the construction of the Spatial VaR using S&P 500 data from 01/01/2008 to 31/12/2008. The abscissa provides the "p"-s at which the VaR estimate (in ordinate) has been calculated. On the left, we present a truncated axis presenting the "q"-s. Here the Spatial VaR tells us in which range the 97th percentile of the log returns of the S&P500 is located. For an intuitive understanding of our approach, note that the 98th percentile of the distribution considered is included in the CI obtained for the estimate of the VaR at 96%.



- For a sequence of p, $p_1 < p_2 < ... < p_k$, we obtain spectrum of VaR_{p_i}, $i = 1, ..., k$. For each VaR_{p_i}, we can build around this value a confidence interval CI_{q_i,p_i}, for a given q_i, $i = 1, ..., k$.
- The parameters q_i and p_i can be equal or different. Now, we consider the area between each VaR_{p_i} and the upper bound of its corresponding CI_{q_i,p_i}.
- This area - delineated between VaR_{p_i} and the upper bound of CI_{q_i} - corresponds to the Spectral Stress VaR measure we propose to use as alert indicator. Indeed, having the VaR for different p provides us with the spectral VaR (SVaR). The construction of a set of confidence intervals around the SVaR provides us with an acceptable range of variation for the VaR_{p_i}.

Conclusions

- **The problem should be discussed in its entirety:**
  - **Risk Measure, Distribution, Estimation, Numerical error, level of confidence should be treated as a single polymorphic organism**
- **Complete mis-alignement between Risk Management and Capital Calculations**
  - **Capital calculation: buffer to face materialisation of risk- therefore we assume it happened, the risk measure is a limit**
  - **Risk Management: try to prevent and mitigate, therefore the risk measure represents an exposure**
- **The wrong regulation leads to a dreadful systemic risk:**
  - **All the bank adopting the same methodology leads in case of failure to a domino effect**
  - **The current regulation prevents the construction of a hollistic approach**